Multiplying Surds

When we multiply surds we combine the surd and then try to simplify if it is possible

Example

$$\sqrt{20} \times \sqrt{15}$$

$$\sqrt{20 \times 15}$$

$$\sqrt{300}$$

$$\sqrt{100 \times 3}$$

$$\sqrt{100} \times \sqrt{3}$$

$$10\sqrt{3}$$

Example

Simplify

$$2\sqrt{10} \times 3\sqrt{6}$$

Method 1

$$2 \times 3 \times \sqrt{10} \times \sqrt{6}$$

$$6\sqrt{10 \times 6}$$

$$6\sqrt{60}$$

$$6\sqrt{4 \times 15}$$

$$6\sqrt{4} \times \sqrt{15}$$

$$12\sqrt{15}$$

Method 2

$$\sqrt{4 \times 10} \times \sqrt{9 \times 6}$$

$$\sqrt{40} \times \sqrt{54}$$

$$\sqrt{2160}$$

$$\sqrt{144 \times 15}$$

$$\sqrt{144} \times \sqrt{15}$$

$$12\sqrt{15}$$

Exercise 1.

Multiply these surds and then simplify where possible. Write the answers in the form $b\sqrt{c}$ where possible.

1) $\sqrt{20} \times \sqrt{32}$

2) $\sqrt{15} \times \sqrt{27}$

3) $\sqrt{10} \times \sqrt{28}$

4) $\sqrt{32} \times \sqrt{3}$

5) $\sqrt{12} \times \sqrt{20}$

6) $\sqrt{6} \times \sqrt{30}$

7) $\sqrt{6} \times \sqrt{8}$

8) $\sqrt{6} \times \sqrt{12}$

9) $\sqrt{18} \times \sqrt{3}$

10) $\sqrt{6} \times \sqrt{15}$

11) $2\sqrt{2} \times 3\sqrt{5}$

12) $2\sqrt{10} \times 2\sqrt{5}$

13) $3\sqrt{3} \times 5\sqrt{2}$

14) $3\sqrt{3} \times 2\sqrt{18}$

15) $2\sqrt{6} \times 3\sqrt{2} \times 2\sqrt{3}$

As we have seen from earlier we can have surds mixed with integers. We would multiply these together in the same way as we would quadratic roots.

Example

Expand and simply

$$\left(2+\sqrt{3}\right)\left(5+\sqrt{3}\right)$$

$$2\left(5+\sqrt{3}\right)+\sqrt{3}\left(5+\sqrt{3}\right)$$

$$10+2\sqrt{3}+5\sqrt{3}+\sqrt{3}\sqrt{3}$$

$$10+7\sqrt{3}+3$$

$$13+7\sqrt{3}$$

Example 2

Expand and simplify

$$\left(2-\sqrt{6}\right)\left(5+\sqrt{10}\right)$$

$$2\left(5+\sqrt{10}\right)-\sqrt{6}\left(5+\sqrt{10}\right)$$

$$10+2\sqrt{10}-5\sqrt{6}-\sqrt{6}\sqrt{10}$$

$$10+2\sqrt{10}-5\sqrt{6}-\sqrt{60}$$

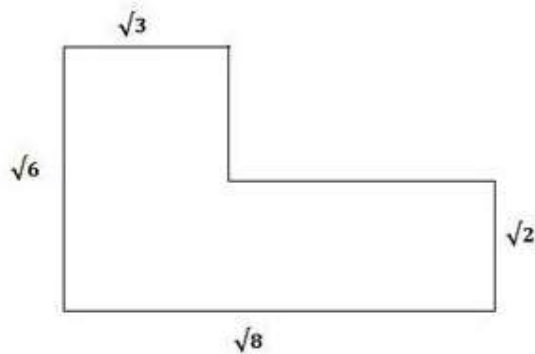$$10+2\sqrt{10}-5\sqrt{6}-2\sqrt{15}$$

Exercise 2

Write these in the form $a + b\sqrt{c}$ where possible, where a and b are integers. (Some may only simplify to $a + b\sqrt{c} + d\sqrt{e} + \ldots$)

1) $\left(2 + \sqrt{6}\right)\left(4 + \sqrt{6}\right)$

2) $\left(3 + \sqrt{10}\right)\left(5 + \sqrt{10}\right)$

3) $\left(4 + \sqrt{2}\right)\left(6 + \sqrt{2}\right)$

4) $\left(5 + \sqrt{3}\right)\left(7 - \sqrt{3}\right)$

5) $\left(5 + \sqrt{5}\right)\left(8 - \sqrt{5}\right)$

6) $\left(2 - \sqrt{7}\right)\left(3 + \sqrt{7}\right)$

7) $\left(4 - \sqrt{11}\right)\left(9 + \sqrt{11}\right)$

8) $\left(3 - \sqrt{5}\right)\left(7 - \sqrt{5}\right)$

9) $\left(2 - \sqrt{3}\right)\left(5 - \sqrt{3}\right)$

10) $\left(2 + \sqrt{3}\right)^2$

11) $\left(2 - \sqrt{3}\right)^2$

12) $\left(2 + \sqrt{5}\right)\left(4 + \sqrt{6}\right)$

13) $\left(3 + \sqrt{5}\right)\left(5 + \sqrt{10}\right)$

14) $\left(4 + \sqrt{2}\right)\left(6 + \sqrt{12}\right)$

15) $\left(5 + \sqrt{3}\right)\left(7 - \sqrt{12}\right)$

16) $\left(5 + \sqrt{15}\right)\left(8 - \sqrt{5}\right)$

17) $\left(2 - \sqrt{7}\right)\left(3 + \sqrt{2}\right)$

18) $\left(4 - \sqrt{3}\right)\left(9 + \sqrt{11}\right)$

19) $\left(3 - \sqrt{3}\right)\left(7 - \sqrt{5}\right)$

20) $\left(2 - \sqrt{8}\right)\left(5 - \sqrt{12}\right)$

Exercise 3

What is the area of this shape? Write your answer as simply as possible.

Answers Exercise 1

1) $8\sqrt{10}$
2) $9\sqrt{5}$
3) $2\sqrt{70}$
4) $4\sqrt{6}$
5) $4\sqrt{15}$
6) $6\sqrt{5}$
7) $4\sqrt{3}$
8) $6\sqrt{2}$
9) $3\sqrt{6}$
10) $3\sqrt{10}$
11) $6\sqrt{10}$
12) $20\sqrt{2}$
13) $15\sqrt{6}$
14) $18\sqrt{6}$
15) 72

Answers Exercise 2

1) $14 + 6\sqrt{6}$
2) $25 + 8\sqrt{10}$
3) $26 + 10\sqrt{2}$
4) $32 + 2\sqrt{3}$
5) $35 + 3\sqrt{5}$
6) $-1 - \sqrt{7}$
7) $25 - 5\sqrt{11}$
8) $26 - 10\sqrt{5}$
9) $13 - 7\sqrt{3}$
10) $7 + 4\sqrt{3}$
11) $7 - 4\sqrt{3}$
12) $8 + 4\sqrt{5} + 2\sqrt{6} + \sqrt{30}$
13) $15 + 5\sqrt{2} + 5\sqrt{5} + 3\sqrt{10}$

14) $24 + 6\sqrt{2} + 8\sqrt{3} + 2\sqrt{6}$
15) $29 - 3\sqrt{3}$
16) $40 - 5\sqrt{3} - 5\sqrt{5} + 8\sqrt{15}$
17) $6 + 2\sqrt{2} - 3\sqrt{7} - \sqrt{14}$
18) $36 - 9\sqrt{3} + 4\sqrt{11} - \sqrt{33}$
19) $21 - \sqrt{3} - \sqrt{5} + \sqrt{15}$
20) $10 - 10\sqrt{2} - 4\sqrt{3} + 4\sqrt{6}$

Answer Exercise 3

$4 + 3\sqrt{2} - \sqrt{6}$